

LEZIONI PRIVATE 346/3103392

$\sqrt[m]{1} = 0 \Leftrightarrow U^m = 1$
 $U = r(\cos \alpha + i \sin \alpha)$



$[r(\cos \alpha + i \sin \alpha)]^m = \cos \theta + i \sin \theta$
 $r^m(\cos m\alpha + i \sin m\alpha) = (\cos \theta + i \sin \theta)$

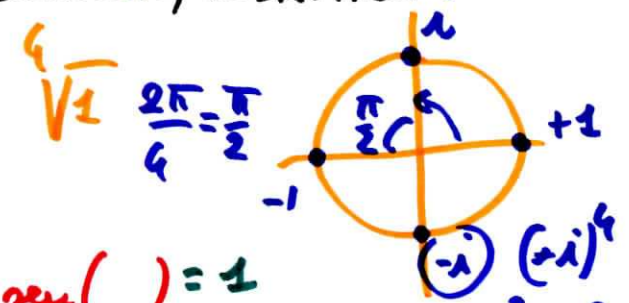
$\begin{cases} r^m = 1 \Rightarrow r = 1 \\ m\alpha = 0 + 2k\pi \end{cases} \begin{cases} r = 1 \\ \alpha = \frac{2k\pi}{m} \end{cases}$
 $k \in \mathbb{Z}$

$U = \cos \frac{2k\pi}{m} + i \sin \frac{2k\pi}{m}, k \in \mathbb{Z}$

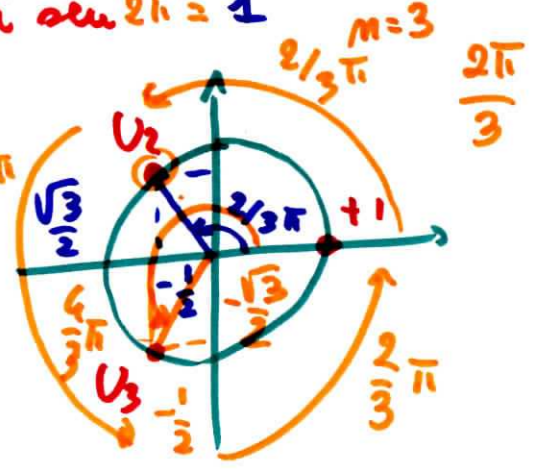
$m=2$
 $\sqrt[2]{1} = \pm 1$
 $k=0 \cos 0 + i \sin 0 = 1$
 $k=1 \cos \frac{2 \cdot 1 \cdot \pi}{2} + i \sin \frac{2 \cdot 1 \cdot \pi}{2} = -1$
 $k=2 \cos \frac{2 \cdot 2 \cdot \pi}{2} + i \sin \frac{2 \cdot 2 \cdot \pi}{2} = 1$
 $k=3 \cos \frac{2 \cdot 3 \cdot \pi}{2} + i \sin \frac{2 \cdot 3 \cdot \pi}{2} = -1$
 $k=-1 \cos \frac{2 \cdot (-1) \cdot \pi}{2} + i \sin \frac{2 \cdot (-1) \cdot \pi}{2} = -1$

$\sqrt[3]{1} \quad m=3$

$k=0 \quad U_2 = \cos \frac{2 \cdot 0 \cdot \pi}{3} + i \sin \frac{2 \cdot 0 \cdot \pi}{3} = 1$
 $k=1 \quad U_2 = \cos \frac{2 \cdot 1 \cdot \pi}{3} + i \sin \frac{2 \cdot 1 \cdot \pi}{3} = -\frac{1}{2} + i \frac{\sqrt{3}}{2}$
 $k=2 \quad U_3 = \cos \frac{2 \cdot 2 \cdot \pi}{3} + i \sin \frac{2 \cdot 2 \cdot \pi}{3} = -\frac{1}{2} - i \frac{\sqrt{3}}{2}$
 $k=3 \quad U_4 = \cos \frac{2 \cdot 3 \cdot \pi}{3} + i \sin \frac{2 \cdot 3 \cdot \pi}{3} = 1$



$U_2^2 = \left[-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right]^2 = \frac{1}{4} + 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} i + \left(-\frac{3}{4} \right) = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$



$U_2^2 \cdot U_3 = U_2^3$
 $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i \right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right) = \left(-\frac{1}{2} \right)^2 - \left(\frac{\sqrt{3}}{2} i \right)^2 = \frac{1}{4} - \left(\frac{3}{4} (-1) \right) = \frac{1}{4} + \frac{3}{4} = 1$